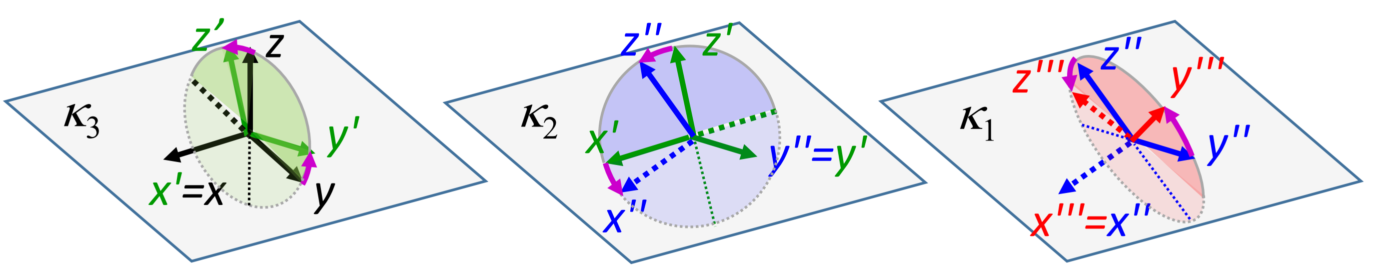
Euler: X→Y→X (fixed axes)   Matrix product: $R = R_x(\kappa_3)\, R_y(\kappa_2)\, R_x(\kappa_1)$

Interpretation (rotating the object) :

Fixed axes :
- *first rotation by* κ_1 *about X*
- *second rotation by* κ_2 *about Y*
- *third rotation by* κ_3 *about X*

Moving axes :
- *first rotation by* κ_3 *about X*
- *second rotation by* κ_2 *about new Y*
- *third rotation by* κ_1 *about new X*

Rotations in terms of moving axes associated with the object :



Rotation matrix R :

$$
\begin{pmatrix}
\cos\kappa_2 & \sin\kappa_1 \sin\kappa_2 & \cos\kappa_1 \sin\kappa_2 \\
\sin\kappa_2 \sin\kappa_3 & -\sin\kappa_1 \cos\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 & -\cos\kappa_1 \cos\kappa_2 \sin\kappa_3 - \sin\kappa_1 \cos\kappa_3 \\
-\sin\kappa_2 \cos\kappa_3 & \sin\kappa_1 \cos\kappa_2 \cos\kappa_3 + \cos\kappa_1 \sin\kappa_3 & \cos\kappa_1 \cos\kappa_2 \cos\kappa_3 - \sin\kappa_1 \sin\kappa_3
\end{pmatrix}
$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$0 \le \kappa_1 < 2\pi$

$0 \le \kappa_2 \le \pi$

$0 \le \kappa_3 < 2\pi$

$\kappa_2 = \arccos(r_{11})$

If $\sin\kappa_2 = 0$: $\kappa_1 = \mathrm{atan2}(-r_{23}, r_{22})$, $\kappa_3 = 0$

If $\sin\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(r_{12}, r_{13})$, $\kappa_3 = \mathrm{atan2}(r_{21}, -r_{31})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = 2\pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

Euler: Y→Z→Y (fixed axes)   Matrix product: $R = R_y(\kappa_3)\, R_z(\kappa_2)\, R_y(\kappa_1)$

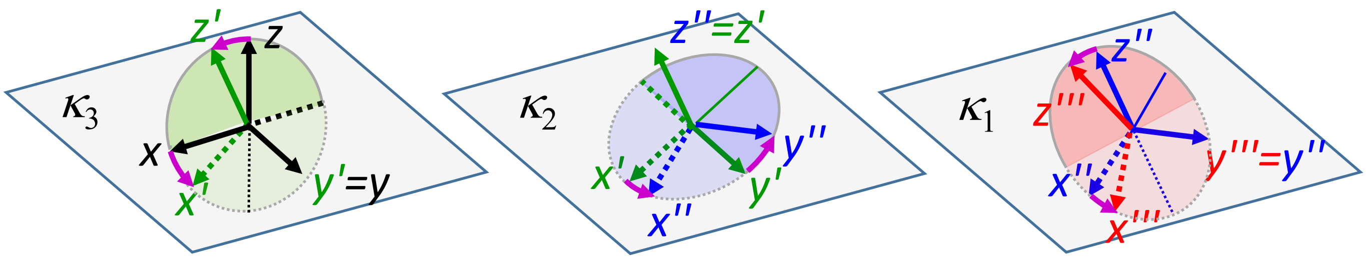
Interpretation (rotating the object) :

Fixed axes :
- first rotation by κ_1 about Y
- second rotation by κ_2 about Z
- third rotation by κ_3 about Y

Moving axes :
- first rotation by κ_3 about Y
- second rotation by κ_2 about new Z
- third rotation by κ_1 about new Y

Rotations in terms of <u>moving</u> axes associated with the object :

Rotation 1   Rotation 2   Rotation 3



Rotation matrix R :

$$\begin{pmatrix} \cos\kappa_1 \cos\kappa_2 \cos\kappa_3 - \sin\kappa_1 \sin\kappa_3 & -\sin\kappa_2 \cos\kappa_3 & \sin\kappa_1 \cos\kappa_2 \cos\kappa_3 + \cos\kappa_1 \sin\kappa_3 \\ \cos\kappa_1 \sin\kappa_2 & \cos\kappa_2 & \sin\kappa_1 \sin\kappa_2 \\ -\cos\kappa_1 \cos\kappa_2 \sin\kappa_3 - \sin\kappa_1 \cos\kappa_3 & \sin\kappa_2 \sin\kappa_3 & -\sin\kappa_1 \cos\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 \end{pmatrix}$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$0 \le \kappa_1 < 2\pi$

$0 \le \kappa_2 \le \pi$

$0 \le \kappa_3 < 2\pi$

$\kappa_2 = \arccos(r_{22})$

If $\sin\kappa_2 = 0$: $\kappa_1 = \mathrm{atan2}(-r_{31}, r_{33})$, $\kappa_3 = 0$

If $\sin\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(r_{23}, r_{21})$, $\kappa_3 = \mathrm{atan2}(r_{32}, -r_{12})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = 2\pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

Euler: Z→X→Z (fixed axes) *Matrix product: $R = R_z(\kappa_3)\, R_x(\kappa_2)\, R_z(\kappa_1)$*

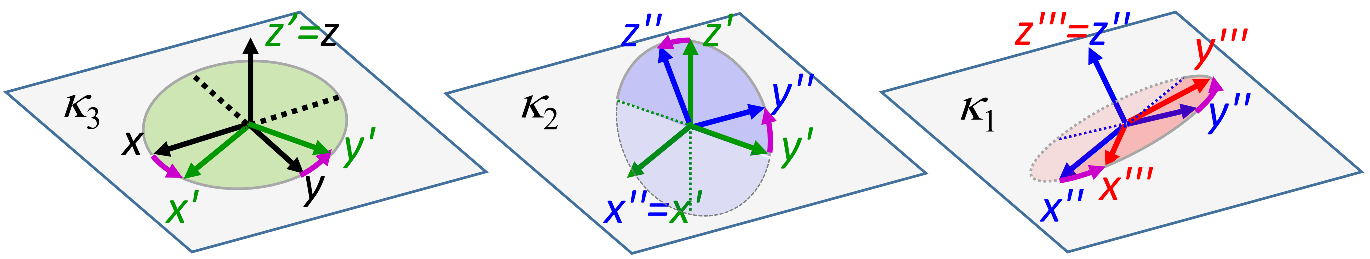
Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about Z*
- *second rotation by κ_2 about X*
- *third rotation by κ_3 about Z*

Moving axes :
- *first rotation by κ_3 about Z*
- *second rotation by κ_2 about new X*
- *third rotation by κ_1 about new Z*

Rotations in terms of <u>moving</u> axes associated with the object :

Rotation 1 *Rotation 2* *Rotation 3*



Rotation matrix R :

$$\begin{pmatrix} -\sin\kappa_1 \cos\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 & -\cos\kappa_1 \cos\kappa_2 \sin\kappa_3 - \sin\kappa_1 \cos\kappa_3 & \sin\kappa_2 \sin\kappa_3 \\ \sin\kappa_1 \cos\kappa_2 \cos\kappa_3 + \cos\kappa_1 \sin\kappa_3 & \cos\kappa_1 \cos\kappa_2 \cos\kappa_3 - \sin\kappa_1 \sin\kappa_3 & -\sin\kappa_2 \cos\kappa_3 \\ \sin\kappa_1 \sin\kappa_2 & \cos\kappa_1 \sin\kappa_2 & \cos\kappa_2 \end{pmatrix}$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$0 \le \kappa_1 < 2\pi$

$0 \le \kappa_2 \le \pi$

$0 \le \kappa_3 < 2\pi$

$\kappa_2 = \arccos(r_{33})$

If $\sin\kappa_2 = 0$: $\kappa_1 = \operatorname{atan2}(-r_{12}, r_{11})$, $\kappa_3 = 0$

If $\sin\kappa_2 > 0$: $\kappa_1 = \operatorname{atan2}(r_{31}, r_{32})$, $\kappa_3 = \operatorname{atan2}(r_{13}, -r_{23})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = 2\pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

Euler: X→Z→X (fixed axes) *Matrix product: $R = R_x(\kappa_3)\, R_z(\kappa_2)\, R_x(\kappa_1)$*

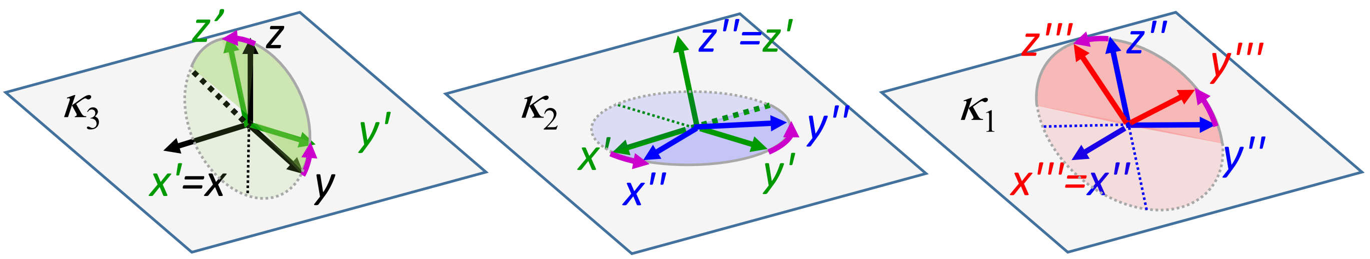
Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about X*
- *second rotation by κ_2 about Z*
- *third rotation by κ_3 about X*

Moving axes :
- *first rotation by κ_3 about X*
- *second rotation by κ_2 about new Z*
- *third rotation by κ_1 about new X*

Rotations in terms of <u>moving</u> axes associated with the object :

Rotation 1 *Rotation 2* *Rotation 3*



Rotation matrix R :

$$\begin{pmatrix} \cos\kappa_2 & -\cos\kappa_1 \sin\kappa_2 & \sin\kappa_1 \sin\kappa_2 \\ \sin\kappa_2 \cos\kappa_3 & \cos\kappa_1 \cos\kappa_2 \cos\kappa_3 - \sin\kappa_1 \sin\kappa_3 & -\sin\kappa_1 \cos\kappa_2 \cos\kappa_3 - \cos\kappa_1 \sin\kappa_3 \\ \sin\kappa_2 \sin\kappa_3 & \cos\kappa_1 \cos\kappa_2 \sin\kappa_3 + \sin\kappa_1 \cos\kappa_3 & -\sin\kappa_1 \cos\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 \end{pmatrix}$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$0 \le \kappa_1 < 2\pi$

$0 \le \kappa_2 \le \pi$

$0 \le \kappa_3 < 2\pi$

$\kappa_2 = \arccos(r_{11})$

If $\sin\kappa_2 = 0$: $\kappa_1 = \mathrm{atan2}(r_{32}, r_{33})$, $\kappa_3 = 0$

If $\sin\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(r_{13}, -r_{12})$, $\kappa_3 = \mathrm{atan2}(r_{31}, r_{21})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = 2\pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

Euler: Y→X→Y (fixed axes)   *Matrix product: $R = R_y(\kappa_3)\, R_x(\kappa_2)\, R_y(\kappa_1)$*

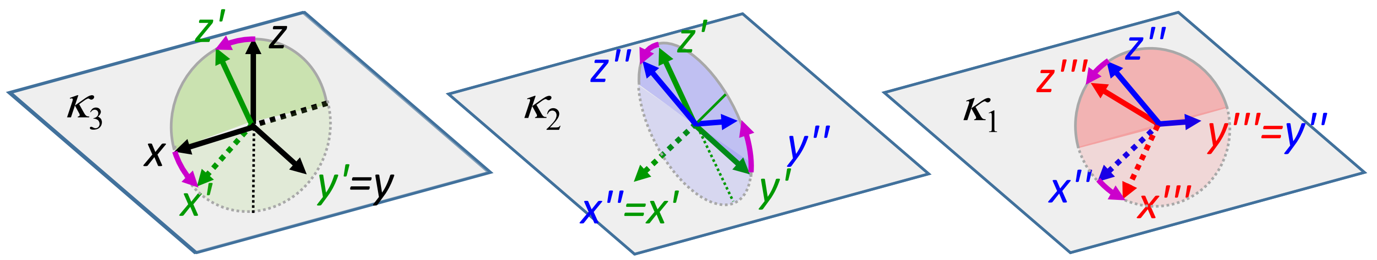
Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about Y*
- *second rotation by κ_2 about X*
- *third rotation by κ_3 about Y*

Moving axes :
- *first rotation by κ_3 about Y*
- *second rotation by κ_2 about new X*
- *third rotation by κ_1 about new Y*

Rotations in terms of <u>moving</u> axes associated with the object :

Rotation 1   *Rotation 2*   *Rotation 3*



Rotation matrix R :

$$R = \begin{pmatrix} -\sin\kappa_1 \cos\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 & \sin\kappa_2 \sin\kappa_3 & \cos\kappa_1 \cos\kappa_2 \sin\kappa_3 + \sin\kappa_1 \cos\kappa_3 \\ \sin\kappa_1 \sin\kappa_2 & \cos\kappa_2 & -\cos\kappa_1 \sin\kappa_2 \\ -\sin\kappa_1 \cos\kappa_2 \cos\kappa_3 - \cos\kappa_1 \sin\kappa_3 & \sin\kappa_2 \cos\kappa_3 & \cos\kappa_1 \cos\kappa_2 \cos\kappa_3 - \sin\kappa_1 \sin\kappa_3 \end{pmatrix}$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$0 \le \kappa_1 < 2\pi$

$0 \le \kappa_2 \le \pi$

$0 \le \kappa_3 < 2\pi$

$\kappa_2 = \arccos(r_{22})$

If $\sin\kappa_2 = 0$: $\kappa_1 = \mathrm{atan2}(r_{13}, r_{11}),\ \kappa_3 = 0$

If $\sin\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(r_{21}, -r_{23}),\ \kappa_3 = \mathrm{atan2}(r_{12}, r_{32})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi\ ;\ \kappa_2' = 2\pi - \kappa_2\ ;\ \kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

Euler: Z→Y→Z (fixed axes) *Matrix product: $R = R_z(\kappa_3)\, R_y(\kappa_2)\, R_z(\kappa_1)$*

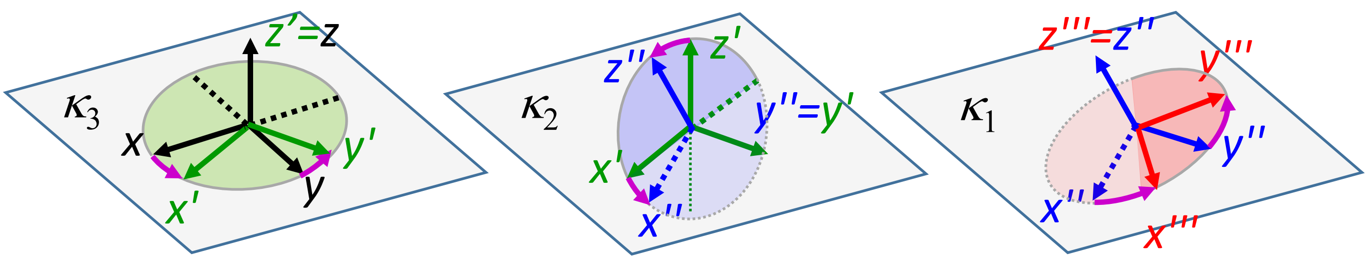
Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about Z*
- *second rotation by κ_2 about Y*
- *third rotation by κ_3 about Z*

Moving axes :
- *first rotation by κ_3 about Z*
- *second rotation by κ_2 about new Y*
- *third rotation by κ_1 about new Z*

Rotations in terms of moving axes associated with the object :

Rotation 1 *Rotation 2* *Rotation 3*



Rotation matrix R :

$$
\begin{pmatrix}
\cos\kappa_1 \cos\kappa_2 \cos\kappa_3 - \sin\kappa_1 \sin\kappa_3 & -\sin\kappa_1 \cos\kappa_2 \cos\kappa_3 - \cos\kappa_1 \sin\kappa_3 & \sin\kappa_2 \cos\kappa_3 \\
\cos\kappa_1 \cos\kappa_2 \sin\kappa_3 + \sin\kappa_1 \cos\kappa_3 & -\sin\kappa_1 \cos\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 & \sin\kappa_2 \sin\kappa_3 \\
-\cos\kappa_1 \sin\kappa_2 & \sin\kappa_1 \sin\kappa_2 & \cos\kappa_2
\end{pmatrix}
$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$0 \le \kappa_1 < 2\pi$

$0 \le \kappa_2 \le \pi$

$0 \le \kappa_3 < 2\pi$

$\kappa_2 = \arccos(r_{33})$

If $\sin\kappa_2 = 0$: $\kappa_1 = \operatorname{atan2}(r_{21}, r_{22})$, $\kappa_3 = 0$

If $\sin\kappa_2 > 0$: $\kappa_1 = \operatorname{atan2}(r_{32}, -r_{31})$, $\kappa_3 = \operatorname{atan2}(r_{23}, r_{13})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = 2\pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

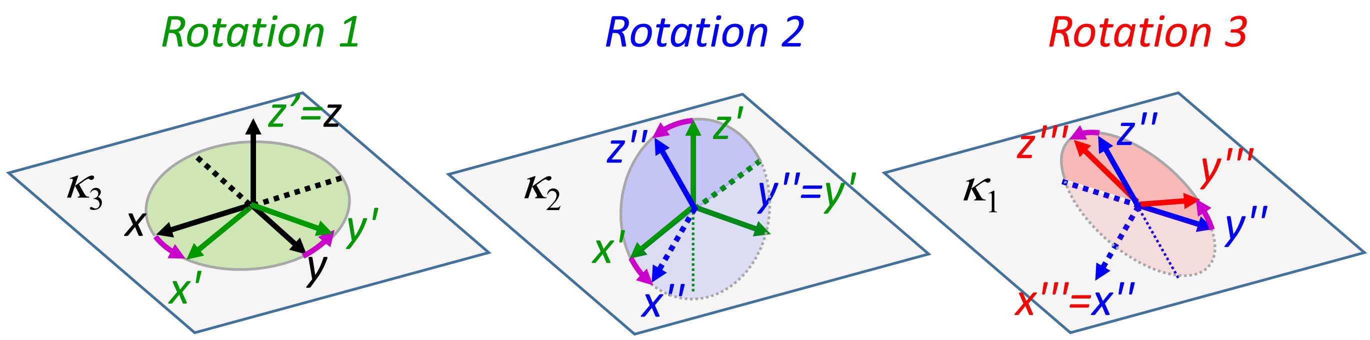
Euler: X→Y→Z (fixed axes) *Matrix product: $R = R_z(\kappa_3)\, R_y(\kappa_2)\, R_x(\kappa_1)$*

Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about X*
- *second rotation by κ_2 about Y*
- *third rotation by κ_3 about Z*

Moving axes :
- *first rotation by κ_3 about Z*
- *second rotation by κ_2 about new Y*
- *third rotation by κ_1 about new X*

Rotations in terms of moving axes associated with the object :



Rotation matrix R :

$$\begin{pmatrix} \cos\kappa_2 \cos\kappa_3 & \sin\kappa_1 \sin\kappa_2 \cos\kappa_3 - \cos\kappa_1 \sin\kappa_3 & \cos\kappa_1 \sin\kappa_2 \cos\kappa_3 + \sin\kappa_1 \sin\kappa_3 \\ \cos\kappa_2 \sin\kappa_3 & \sin\kappa_1 \sin\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 & \cos\kappa_1 \sin\kappa_2 \sin\kappa_3 - \sin\kappa_1 \cos\kappa_3 \\ -\sin\kappa_2 & \sin\kappa_1 \cos\kappa_2 & \cos\kappa_1 \cos\kappa_2 \end{pmatrix}$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$$0 \le \kappa_1 < 2\pi$$
$$-\tfrac{1}{2}\pi \le \kappa_2 \le \tfrac{1}{2}\pi$$
$$0 \le \kappa_3 < 2\pi$$

$\kappa_2 = -\arcsin(r_{31})$

If $\cos\kappa_2 = 0$: $\kappa_1 = \operatorname{atan2}(-r_{23}, r_{22})$, $\kappa_3 = 0$

If $\cos\kappa_2 > 0$: $\kappa_1 = \operatorname{atan2}(r_{32}, r_{33})$, $\kappa_3 = \operatorname{atan2}(r_{21}, r_{11})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi\,;\ \kappa_2' = \pi - \kappa_2\,;\ \kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

Euler: Y→Z→X (fixed axes) *Matrix product: $R = R_x(\kappa_3)\ R_z(\kappa_2)\ R_y(\kappa_1)$*

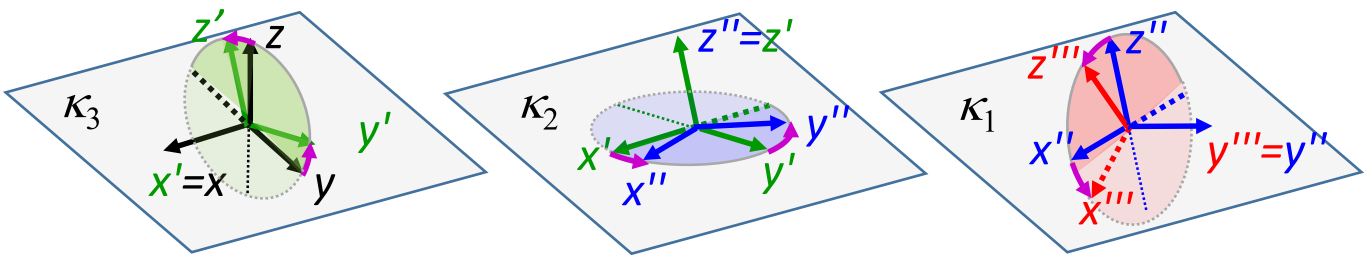
Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about Y*
- *second rotation by κ_2 about Z*
- *third rotation by κ_3 about X*

Moving axes :
- *first rotation by κ_3 about X*
- *second rotation by κ_2 about new Z*
- *third rotation by κ_1 about new Y*

Rotations in terms of <u>moving</u> axes associated with the object :

Rotation 1 *Rotation 2* *Rotation 3*



Rotation matrix R :

$$
\begin{pmatrix}
\cos\kappa_1 \cos\kappa_2 & -\sin\kappa_2 & \sin\kappa_1 \cos\kappa_2 \\
\cos\kappa_1 \sin\kappa_2 \cos\kappa_3 + \sin\kappa_1 \sin\kappa_3 & \cos\kappa_2 \cos\kappa_3 & \sin\kappa_1 \sin\kappa_2 \cos\kappa_3 - \cos\kappa_1 \sin\kappa_3 \\
\cos\kappa_1 \sin\kappa_2 \sin\kappa_3 - \sin\kappa_1 \cos\kappa_3 & \cos\kappa_2 \sin\kappa_3 & \sin\kappa_1 \sin\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3
\end{pmatrix}
$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$$0 \le \kappa_1 < 2\pi$$
$$-\tfrac{1}{2}\pi \le \kappa_2 \le \tfrac{1}{2}\pi$$
$$0 \le \kappa_3 < 2\pi$$

$\kappa_2 = -\arcsin(r_{12})$

If $\cos\kappa_2 = 0$: $\kappa_1 = \mathrm{atan2}(-r_{31}, r_{33})$, $\kappa_3 = 0$

If $\cos\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(r_{13}, r_{11})$, $\kappa_3 = \mathrm{atan2}(r_{32}, r_{22})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = \pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

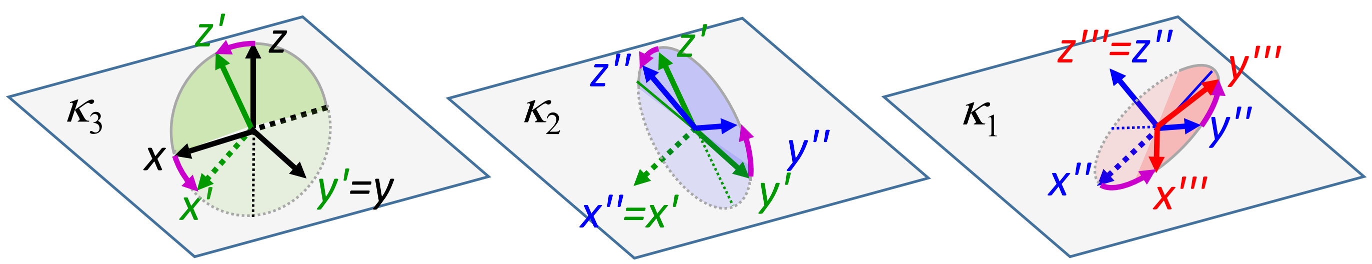
Euler: Z→X→Y (fixed axes) *Matrix product: $R = R_y(\kappa_3)\, R_x(\kappa_2)\, R_z(\kappa_1)$*

Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about Z*
- *second rotation by κ_2 about X*
- *third rotation by κ_3 about Y*

Moving axes :
- *first rotation by κ_3 about Y*
- *second rotation by κ_2 about new X*
- *third rotation by κ_1 about new Z*

Rotations in terms of <u>moving</u> axes associated with the object :



Rotation matrix R :

$$
\begin{pmatrix}
\sin\kappa_1 \sin\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 & \cos\kappa_1 \sin\kappa_2 \sin\kappa_3 - \sin\kappa_1 \cos\kappa_3 & \cos\kappa_2 \sin\kappa_3 \\
\sin\kappa_1 \cos\kappa_2 & \cos\kappa_1 \cos\kappa_2 & -\sin\kappa_2 \\
\sin\kappa_1 \sin\kappa_2 \cos\kappa_3 - \cos\kappa_1 \sin\kappa_3 & \cos\kappa_1 \sin\kappa_2 \cos\kappa_3 + \sin\kappa_1 \sin\kappa_3 & \cos\kappa_2 \cos\kappa_3
\end{pmatrix}
$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$$0 \le \kappa_1 < 2\pi$$
$$-\tfrac{1}{2}\pi \le \kappa_2 \le \tfrac{1}{2}\pi$$
$$0 \le \kappa_3 < 2\pi$$

$\kappa_2 = -\arcsin(r_{23})$

If $\cos\kappa_2 = 0$: $\kappa_1 = \mathrm{atan2}(-r_{12}, r_{11})$, $\kappa_3 = 0$

If $\cos\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(r_{21}, r_{22})$, $\kappa_3 = \mathrm{atan2}(r_{13}, r_{33})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = \pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

Euler: X→Z→Y (fixed axes) **Matrix product: $R = R_y(\kappa_3)\, R_z(\kappa_2)\, R_x(\kappa_1)$**

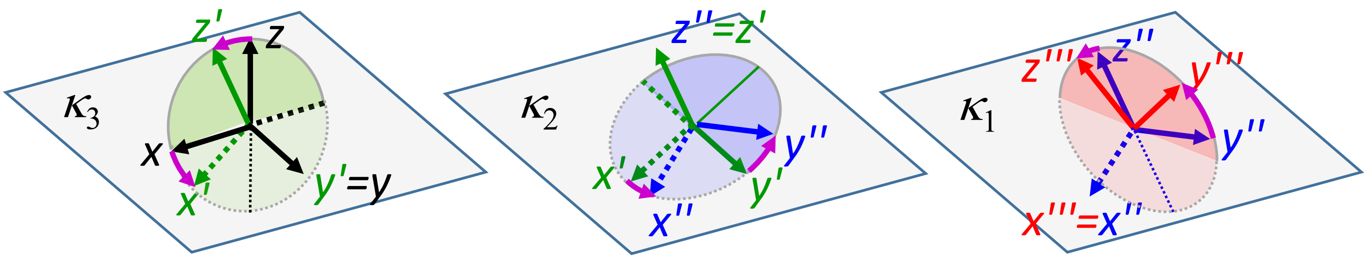
Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about X*
- *second rotation by κ_2 about Z*
- *third rotation by κ_3 about Y*

Moving axes :
- *first rotation by κ_3 about Y*
- *second rotation by κ_2 about new Z*
- *third rotation by κ_1 about new X*

Rotations in terms of moving axes associated with the object :

Rotation 1 *Rotation 2* *Rotation 3*



Rotation matrix R :

$$
\begin{pmatrix}
\cos\kappa_2 \cos\kappa_3 & -\cos\kappa_1 \sin\kappa_2 \cos\kappa_3 + \sin\kappa_1 \sin\kappa_3 & \sin\kappa_1 \sin\kappa_2 \cos\kappa_3 + \cos\kappa_1 \sin\kappa_3 \\
\sin\kappa_2 & \cos\kappa_1 \cos\kappa_2 & -\sin\kappa_1 \cos\kappa_2 \\
-\cos\kappa_2 \sin\kappa_3 & \cos\kappa_1 \sin\kappa_2 \sin\kappa_3 + \sin\kappa_1 \cos\kappa_3 & -\sin\kappa_1 \sin\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3
\end{pmatrix}
$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$$0 \le \kappa_1 < 2\pi$$
$$-\tfrac{1}{2}\pi \le \kappa_2 \le \tfrac{1}{2}\pi$$
$$0 \le \kappa_3 < 2\pi$$

$\kappa_2 = \arcsin(r_{21})$

If $\cos\kappa_2 = 0$: $\kappa_1 = \mathrm{atan2}(r_{32}, r_{33})$, $\kappa_3 = 0$

If $\cos\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(-r_{23}, r_{22})$, $\kappa_3 = \mathrm{atan2}(-r_{31}, r_{11})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = \pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

Euler: Y→X→Z (fixed axes) Matrix product: $R = R_z(\kappa_3)\, R_x(\kappa_2)\, R_y(\kappa_1)$

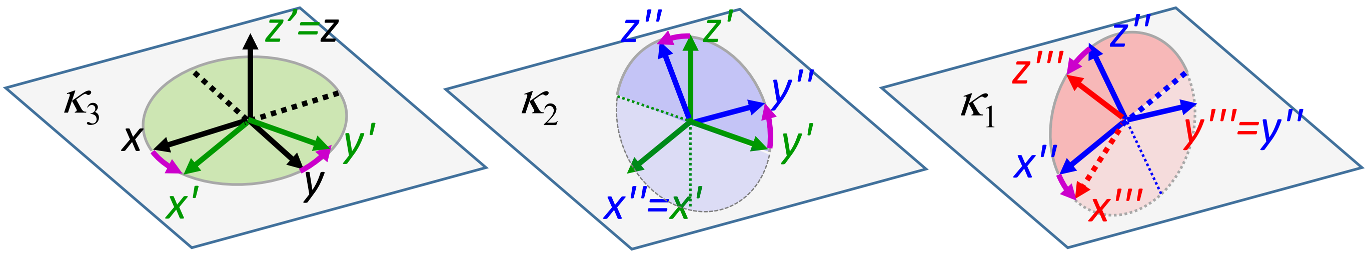
Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about Y*
- *second rotation by κ_2 about X*
- *third rotation by κ_3 about Z*

Moving axes :
- *first rotation by κ_3 about Z*
- *second rotation by κ_2 about new X*
- *third rotation by κ_1 about new Y*

Rotations in terms of moving axes associated with the object :

Rotation 1 *Rotation 2* *Rotation 3*



Rotation matrix R :

$$
\begin{pmatrix}
-\sin\kappa_1 \sin\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 & -\cos\kappa_2 \sin\kappa_3 & \cos\kappa_1 \sin\kappa_2 \sin\kappa_3 + \sin\kappa_1 \cos\kappa_3 \\
\sin\kappa_1 \sin\kappa_2 \cos\kappa_3 + \cos\kappa_1 \sin\kappa_3 & \cos\kappa_2 \cos\kappa_3 & -\cos\kappa_1 \sin\kappa_2 \cos\kappa_3 + \sin\kappa_1 \sin\kappa_3 \\
-\sin\kappa_1 \cos\kappa_2 & \sin\kappa_2 & \cos\kappa_1 \cos\kappa_2
\end{pmatrix}
$$

Getting angles from the matrix $\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

$$0 \le \kappa_1 < 2\pi$$
$$-\tfrac{1}{2}\pi \le \kappa_2 \le \tfrac{1}{2}\pi$$
$$0 \le \kappa_3 < 2\pi$$

$\kappa_2 = \arcsin(r_{32})$

If $\cos\kappa_2 = 0$: $\kappa_1 = \mathrm{atan2}(r_{13}, r_{11})$, $\kappa_3 = 0$

If $\cos\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(-r_{31}, r_{33})$, $\kappa_3 = \mathrm{atan2}(-r_{12}, r_{22})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = \pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a

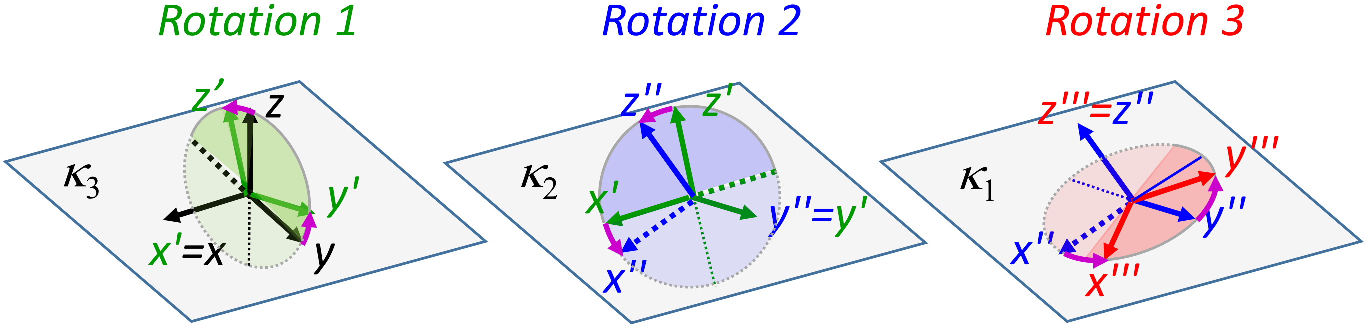
Euler: $Z \rightarrow Y \rightarrow X$ (fixed axes) Matrix product: $R = R_x(\kappa_3)\, R_y(\kappa_2)\, R_z(\kappa_1)$

Interpretation (rotating the object) :

Fixed axes :
- *first rotation by κ_1 about Z*
- *second rotation by κ_2 about Y*
- *third rotation by κ_3 about X*

Moving axes :
- *first rotation by κ_3 about X*
- *second rotation by κ_2 about new Y*
- *third rotation by κ_1 about new Z*

Rotations in terms of moving axes associated with the object :



Rotation matrix R :

$$\begin{pmatrix} \cos\kappa_1 \cos\kappa_2 & -\sin\kappa_1 \cos\kappa_2 & \sin\kappa_2 \\ \cos\kappa_1 \sin\kappa_2 \sin\kappa_3 + \sin\kappa_1 \cos\kappa_3 & -\sin\kappa_1 \sin\kappa_2 \sin\kappa_3 + \cos\kappa_1 \cos\kappa_3 & -\cos\kappa_2 \sin\kappa_3 \\ -\cos\kappa_1 \sin\kappa_2 \cos\kappa_3 + \sin\kappa_1 \sin\kappa_3 & \sin\kappa_1 \sin\kappa_2 \cos\kappa_3 + \cos\kappa_1 \sin\kappa_3 & \cos\kappa_2 \cos\kappa_3 \end{pmatrix}$$

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If $\cos\kappa_2 > 0$: $\kappa_1 = \mathrm{atan2}(-r_{12}, r_{11})$, $\kappa_3 = \mathrm{atan2}(-r_{23}, r_{33})$

Equivalent angles : $\kappa_1' = \kappa_1 + \pi$; $\kappa_2' = \pi - \kappa_2$; $\kappa_3' = \kappa_3 + \pi$

Relevant software : n/a